Algebraic Methods- Questions

June 2019 Mathematics Advanced Paper 1: Pure Mathematics 1

1.

(i) Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4

(4)

(ii) "Given $x \in \mathbb{R}$, the value of $|3x - 28|$ is greater than or equal to the value of $(x - 9)$."
State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

May 2019 Mathematics Advanced Paper 1: Pure Mathematics 1

2.

$$f(x) = 2x^3 - 13x^2 + 8x + 48$$

(a) Prove that $(x - 4)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, show that the equation $f(x) = 0$ has only two distinct roots.

(4)

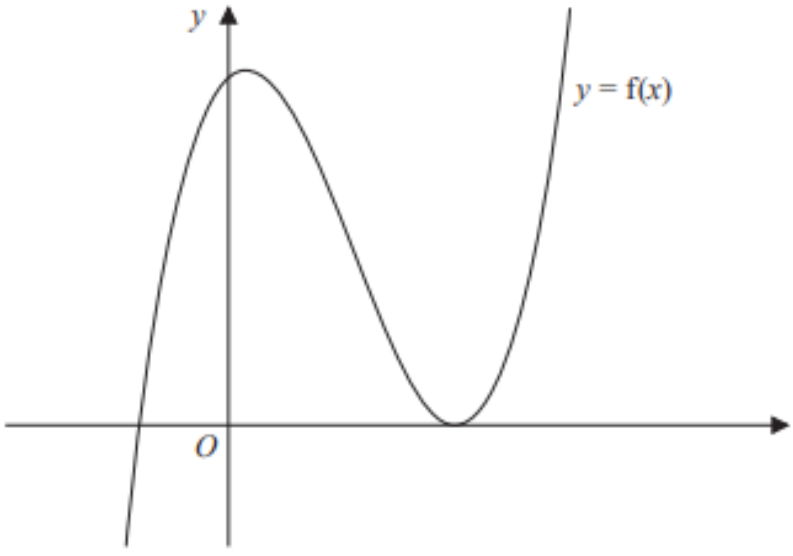

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$.

3.

Given $n \in \mathbb{N}$, prove that $n^3 + 2$ is not divisible by 8

(4)

May 2018 Mathematics Advanced Paper 1: Pure Mathematics 1

4.

$$g(x) = 4x^3 - 12x^2 - 15x + 50$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $g(x)$.

(2)

(b) Hence show that $g(x)$ can be written in the form $g(x) = (x + 2)(ax + b)^2$, where a and b are integers to be found.

(4)

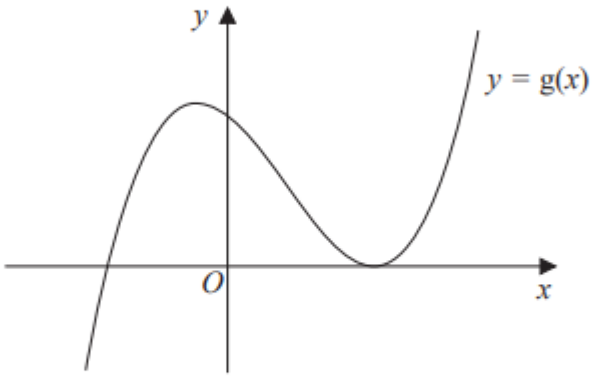

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = g(x)$

(c) Use your answer to part (b), and the sketch, to deduce the values of x for which

(i) $g(x) \leqslant 0$

(ii) $g(2x) = 0$

(3)

May 2017 Mathematics Advanced Paper 1: Pure Mathematics 2

5.

6. $f(x) = -6x^3 - 7x^2 + 40x + 21$

(*a*) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$

(2)

(*b*) Factorise $f(x)$ completely.

(4)

(*c*) Hence solve the equation

$$6(2^{3y}) + 7(2^{2y}) = 40(2^y) + 21$$

giving your answer to 2 decimal places.

(3)

May 2016 Mathematics Advanced Paper 1: Pure Mathematics 2

6.

4. $f(x) = 6x^3 + 13x^2 - 4$

(*a*) Use the remainder theorem to find the remainder when $f(x)$ is divided by $(2x + 3)$.

(2)

(*b*) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$.

(2)

(*c*) Factorise $f(x)$ completely.

(4)

May 2015 Mathematics Advanced Paper 1: Pure Mathematics 2

7.

3. $f(x) = 6x^3 + 3x^2 + Ax + B$, where A and B are constants.

Given that when $f(x)$ is divided by $(x + 1)$ the remainder is 45,

(*a*) show that $B - A = 48$.

(2)

Given also that $(2x + 1)$ is a factor of $f(x)$,

(*b*) find the value of A and the value of B.

(4)

(*c*) Factorise $f(x)$ fully.

(3)

May 2014 Mathematics Advanced Paper 1: Pure Mathematics 2

8.

2. $f(x) = 2x^3 - 7x^2 + 4x + 4.$

(*a*) Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$.

(2)

(*b*) Factorise $f(x)$ completely.

(4)

May 2013 Mathematics Advanced Paper 1: Pure Mathematics 2

9.

3. $f(x) = 2x^3 - 5x^2 + ax + 18$

where a is a constant.

Given that $(x - 3)$ is a factor of $f(x)$,

(*a*) show that $a = -9$,

(2)

(*b*) factorise $f(x)$ completely.

(4)

Given that

$$g(y) = 2(3^{3y}) - 5(3^{2y}) - 9(3^y) + 18,$$

(*c*) find the values of y that satisfy $g(y) = 0$, giving your answers to 2 decimal places where appropriate.

(3)

Jan 2013 Mathematics Advanced Paper 1: Pure Mathematics 2

10.

2. $f(x) = ax^3 + bx^2 - 4x - 3$, where a and b are constants.

Given that $(x - 1)$ is a factor of $f(x)$,

(*a*) show that $a + b = 7$.

(2)

Given also that, when $f(x)$ is divided by $(x + 2)$, the remainder is 9,

(*b*) find the value of a and the value of b, showing each step in your working.

(4)

May 2012 Mathematics Advanced Paper 1: Pure Mathematics 2

11.

4. $f(x) = 2x^3 - 7x^2 - 10x + 24.$

(*a*) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$.

(2)

(*b*) Factorise $f(x)$ completely.

(4)

Jan 2012 Mathematics Advanced Paper 1: Pure Mathematics 2

12.

5. $f(x) = x^3 + ax^2 + bx + 3$, where a and b are constants.

Given that when $f(x)$ is divided by $(x + 2)$ the remainder is 7,

(*a*) show that $2a - b = 6$.

(2)

Given also that when $f(x)$ is divided by $(x - 1)$ the remainder is 4,

(*b*) find the value of a and the value of b.

(4)

May 2011 Mathematics Advanced Paper 1: Pure Mathematics 2

13.

1. $f(x) = 2x^3 - 7x^2 - 5x + 4$

(*a*) Find the remainder when $f(x)$ is divided by $(x - 1)$.

(2)

(*b*) Use the factor theorem to show that $(x + 1)$ is a factor of $f(x)$.

(2)

(*c*) Factorise $f(x)$ completely.

(4)

Jan 2011 Mathematics Advanced Paper 1: Pure Mathematics 2

14.

1. $f(x) = x^4 + x^3 + 2x^2 + ax + b,$

where a and b are constants.

When $f(x)$ is divided by $(x - 1)$, the remainder is 7.

(a) Show that $a + b = 3$.

(2)

When $f(x)$ is divided by $(x + 2)$, the remainder is -8.

(b) Find the value of a and the value of b.

(5)

Jun 2010 Mathematics Advanced Paper 1: Pure Mathematics 2

15.

2. $f(x) = 3x^3 - 5x^2 - 58x + 40.$

(a) Find the remainder when $f(x)$ is divided by $(x - 3)$.

(2)

Given that $(x - 5)$ is a factor of $f(x)$,

(b) find all the solutions of $f(x) = 0$.

(5)

Jan 2010 Mathematics Advanced Paper 1: Pure Mathematics 2

16.

3. $f(x) = 2x^3 + ax^2 + bx - 6,$

where a and b are constants.

When $f(x)$ is divided by $(2x - 1)$ the remainder is -5.

When $f(x)$ is divided by $(x + 2)$ there is no remainder.

(a) Find the value of a and the value of b.

(6)

(b) Factorise $f(x)$ completely.

(3)

June 2017 Mathematics Advanced Paper 1: Pure Mathematics 3

17.

1. Express $\frac{4x}{x^2-9}-\frac{2}{x+3}$ as a single fraction in its simplest form.

(4)

June 2012 Mathematics Advanced Paper 1: Pure Mathematics 3

18.

1. Express

$$\frac{2(3x+2)}{9x^2-4}-\frac{2}{3x+1}$$

as a single fraction in its simplest form.

(4)

Jan 2011 Mathematics Advanced Paper 1: Pure Mathematics 3

19.

2. (*a*) Express

$$\frac{4x-1}{2(x-1)}-\frac{3}{2(x-1)(2x-1)}$$

as a single fraction in its simplest form.

(4)

Given that

$$\mathrm{f}(x)=\frac{4x-1}{2(x-1)}-\frac{3}{2(x-1)(2x-1)}-2, \qquad x>1,$$

(*b*) show that

$$\mathrm{f}(x)=\frac{3}{2x-1}.$$

(2)

(*c*) Hence differentiate $\mathrm{f}(x)$ and find $\mathrm{f}'(2)$.

(3)

Jan 2010 Mathematics Advanced Paper 1: Pure Mathematics 3

20.

1. Express

$$\frac{x+1}{3x^2-3} - \frac{1}{3x+1}$$

as a single fraction in its simplest form.

(4)